Chapter 2
The Great War and the First Triode Designs: Abraham, Bloch, Blondel, Van der Pol

2.1 The Great War and the Rise of Wireless Telegraphy: *The T.M. Valve and the Multivibrator*

2.1.1 General Ferrié: From Wireless Telegraphy to the Eiffel Tower

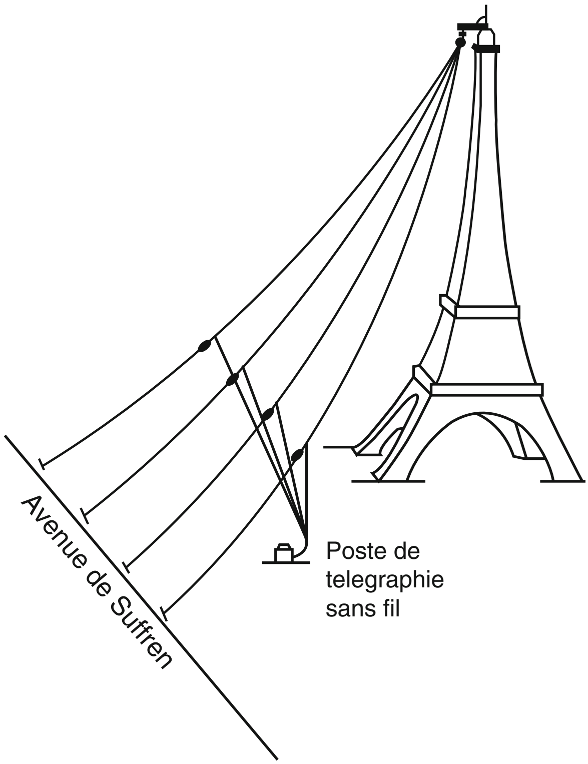
Gustave Ferrié[1] joined the École Polytechnique in 1887 at age 19 and chose l'arme du Génie (engineering) afterwards. He became a radio transmission engineer in 1893, specializing in military telegraphy in 1893. In 1897, he was named Head of the École de Télégraphie Militaire that had been created in 1895 at Mont Valérien. From 1899, the young captain showed an interest in wireless telegraphy after witnessing the first experiments carried out by Guglielmo Marconi on short-distance Hertzian links. The same year, The Minister of War, Charles de Freycinet appointed him to the Committee on Wireless Telegraphy research between France and the United Kingdom in order to write a report on the military applications for this communication medium. In October, with commandant Boulanger, he published the first French study[2] on wireless telegraphy. He improved the coherer designed by Édouard Branly, whose lectures he attended, by perfecting an electrolytic detector in 1900. In autumn 1903, Ferrié met Camille Flammarion and told her about one of the major problems with the development of radio transmission at the time – the size of the antenna. Camille Flammarion, being very close to Gustave Eiffel, asked him if Ferrié could use his 300-m high (986 ft) tower for his radio broadcasting tests. The government soon authorized the building of the first experimental military installation, which was based at Champ de Mars, the tower being used to support the antenna (see Fig. 2.1).

[1] See also *Notice sur les travaux scientifiques et techniques de M. Gustave Ferrié*, Ferrié (1921).

[2] See Boulanger and Ferrié (1899). Chapter V was redacted with Blondel's collaboration.

J.-M. Ginoux, *History of Nonlinear Oscillations Theory in France (1880–1940)*,
Archimedes 49, DOI 10.1007/978-3-319-55239-2_2

Fig. 2.1 The Eiffel tower's first antenna (1903–1908), from Turpain (1908, 242)



This initiative probably saved the Eiffel tower from being razed, by turning it into the cornerstone of the military wireless transmission network. From 1908 to 1914, *Commandant* Ferrié worked on developing mobile radio communication military units (auto-mobile field stations, planes, and dirigible stations). He created the pendulum comparison method by using wireless telegraphy, making it possible to determine the longitude, within a few meters, of any location as long as it is situated within range of the emitting station. Indeed, in 1911, Ferrié started a series of experiments aiming at accurately determining the difference in longitude between Toulon and Paris, and then between Paris and Washington, using radio signals emitted from the Eiffel tower. Soon after, he installed the time signal emitter used by navigators at the top of the Eiffel tower. During this time, he became a corresponding member for the *Bureau des Longitudes* (1911), the *Comité d'Électricité* (1912), and was appointed as a lecturer on the *Cours de Télégraphie Sans Fil* of the *École Supérieure d'Électricité* (1911). Just before the First World War, Ferrié was promoted to the rank of *colonel* and became the technical director of the *Radiotélégraphie Militaire* department, which would become the *Établissement Central du Matériel de la Radiotélégraphie Militaire* (E.C.M.R.) in 1917. Because it was used to listen into enemy communications, the Eiffel Tower, formerly called the "Dame de Fer" ("Iron Lady"), earned a new nickname, "La grande Oreille" ("The big Ear").

It was in this way, thanks to the interception of a German message,[3] that Joffre was informed of the advance of von Klück's troops, and decided to requisition all the taxis in Paris in order to send soldiers to *La Marne*. At the time, radiotelegraphy equipment was used to receive and emit signals and was based on the singing arc concept described in the previous paragraph. In fact, although the audion[4] had been created in 1907 by Lee de Forest, the triode appeared only in the first days of the war, and then in notably curious circumstances as we will see. A key player was the French engineer Paul Pichon. Pichon had actually deserted the French army in 1900 and migrated to Germany, where he earned a living by teaching French. Among his students were the children of Count von Arco, one of the founders of the Telefunken Company, who then hired him as a technical representative. In March 1913, Abraham visited the United States with him and they met with Lee de Forest, which is how he managed to learn about the latest improvements and applications of audions, which could be used as amplifiers and oscillators from then on.[5] In the summer of 1914, he went to the United States on an assignment from the German company Telefunken to try and gather samples of the most recent valves for wireless telegraphy in order to bring them back and test them in Germany. During his stay, Pichon visited the Western Electric Company, where he was given the latest high-vacuum Audion, and was provided with full information on their use. On the way back from his stay in America, the transatlantic ship stopped in London on the third of August 1914, the day Germany declared war on France. Pichon was then considered as a deserter in France and as an alien in Germany. He decided to go to Calais where he was arrested and brought to the French military authorities which were represented by colonel Ferrié. In October 1914, Ferrié gathered a team of specialists whose mission was to develop a French audion, which should be sturdy, have regular characteristics, and be easy to produce industrially.[6]

2.1.2 The T.M. Valve: Télégraphie Militaire

In October 1915, Ferrié decided to send Abraham to Lyon where he ordered a military emitter to be built that would be capable of replacing the one on the Eiffel tower in case it was rendered unusable. At the same time, a 100 kW emitter was about to depart for Saigon, which should have been conveyed by Captain François Péri of the *Infanterie Coloniale*, chief of the Service Radio of Indochina. Ferrié managed the feat of getting the equipment and men escorting it put at his disposal.

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[3]The "emitted free-to-air" message picked up by the Eiffel Tower was the following: "Très bien compris, gagnez rive sud de la Marne. Oberste Heeresleitung (GQG)."

[4]The audion generator was the first triode-type electronic tube. Patent N° 841 386, 15 January 1907.

[5]The audion replaced the singing arc as an emitter, but also a receptor.

[6]See Champeix (1980, 16).

He appointed Péri as chief of the station in Lyon-La-Doua, and the engineer Joseph Bethenod, who had designed this emitter S.F.R.,[7] as head technician. Less than two months later, Péri managed to build barracks and to erect the antenna consisting of 8 towers of 120 m high each, making the radio station of Lyon La Doua operational. Choosing Lyon was not just a strategic move, it was also due to the proximity of the Grammont factory,[8] which produced incandescent valves. The self-taught engineer Jacques Biguet was briefly appointed as director.

Abraham and Péri initially tried to recreate Lee de Forest's audions, but their fragile structure and lack of stability made them unsuitable for military use. After several unsuccessful attempts, Abraham created a fourth structure in December 1914, which was used from February to October 1915 (see Fig. 2.2). A copy of this valve, called the "Abraham lamp" is still in the *Arts et Métiers* museum to

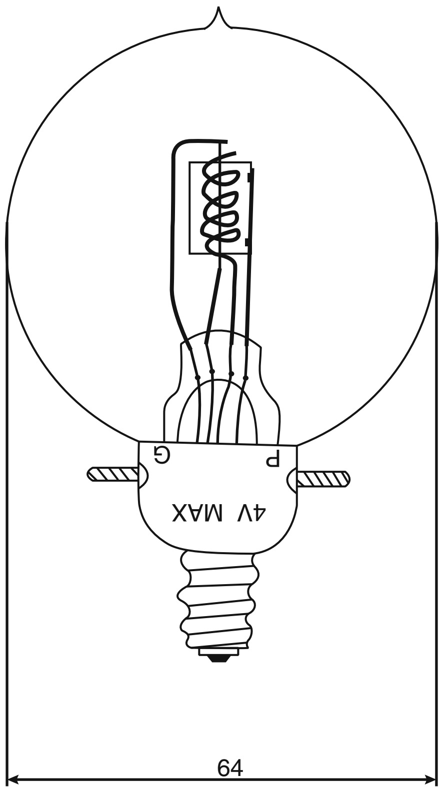

Fig. 2.2 Abraham valve, from Champeix (1980, 15)

[7] Société Française Radioélectrique.

[8] François Grammont, *normalien* like Henri Abraham, was then *Capitaine des Zouaves*. He was demobilized at the beginning of 1915, at the initiative of Ferrié, in order to go back to his post as director of his factory.

this day.[9] It has a cylindrical structure, which appears to have been designed by Abraham. Afterwards he created several new processes for improving the quality of the vacuum inside the lamp, insuring better reliability and stability. However, the relationship between the captain and the physicist soon deteriorated, and a competition arose between them. The atmosphere became so toxic that Ferrié had to call Abraham back to Paris in May 1915. Following his departure, Péri, who possessed extensive skills in the radio-engineering field, resumed his experiments with Biguet to improve the device. He created a valve with a mobile plate and a grid, which made it easier to investigate the characteristics experimentally. This collaboration resulted in the creation the famous T.M. valve (see Figs. 2.3 and 2.4), for which he registered four patents under the names of Mr. Peri and Biguet. The main patent n° 492657 was requested on October 23rd, 1915 and delivered on March 21st, 1919 (Fig. 2.5).

The cylindrical structure of the T.M. valve greatly improved its sturdiness and emission quality. Moreover, the four-pin cap allows quick replacement, as opposed to screw caps (compare Figs. 2.2 and 2.3–2.4). The T.M. valve, also called the "French valve", was refined to such a degree of reliability that it was used by the French and then by the allied armies, and over one million copies were mass-produced over the course of the conflict. It therefore appears that the first triode valve prototype was indeed created by Abraham in December 1914. However, the famous T.M. valve was actually patented by Péri and Biguet in May 1915, after Abraham left. A huge squabble ensued over the invention's paternity. Colonel Ferrié did not forgive Peri for patenting it, as he considered that the credit should have fully gone to Abraham.[10] Hence, when he asked Camille Gutton in March 1918 to write a "Note on the three-electrode valve lamps and their uses[11]", which was published by the *Établissement Central du Matériel de la Radiotélégraphie Militaire* (E.C.M.R.), Péri was not credited as a contributor to the T.M. valve production. This report, n° 412 of E.C.M.R., of more than a hundred and seventy pages and classified as a "military secret", is a remarkable synthesis of the work carried out in France during the First World War, in regard to, on the one hand, the creation of a T.M. lamp, and on the other hand the developing of the multivibrator (see Figs. 2.6, 2.7 and 2.8).

[9]Inventory n° 21204-0000-.

[10]According to Champeix (1980, 20 and following) Abraham refused, in spite of Ferrié's injunctions, to register any patent, and to ask for any compensation.

[11]This E.C.M.R. note, which was destroyed during the rebuilding of the *Service Historique de la Défense* (S.H.D.), was found in a collection: Mr. Jacques Denys's, who agreed to send us a copy. See *infra*.

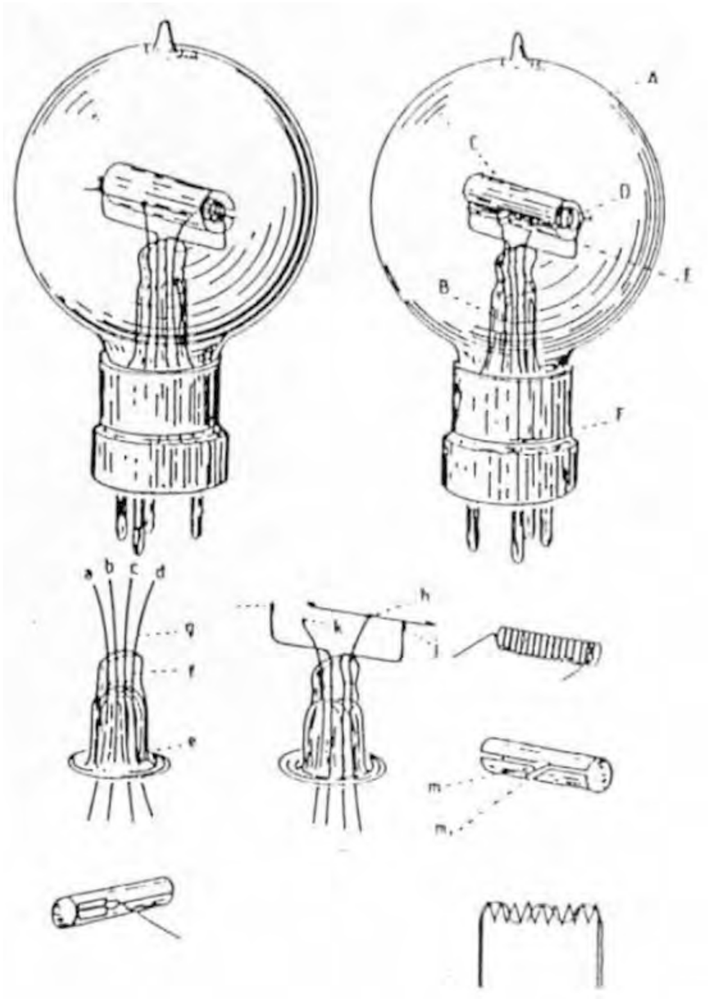

Fig. 2.3 T.M. valve, from Champeix (1980, 18)

On his return to Paris, Abraham joined the Military Telegraphic service's second group, under the supervision of Captain Paul Brenot,[12] along with Maurice de Broglie, Paul Laüt and Lucien Lévy. Abraham then resumed his duties as head of

[12]Paul Brenot (1880–1967) joined the *École Polytechnique* in 1899 and was appointed as attendant to Ferrié (X 1887) from 1904. He played an important part in the development of the *S.F.R.*, created by Joseph Bethenod and Émile Girardeau (X 1902) in 1910, and backed his participation in Military Telegraphy for the construction of high-quality wireless telegraphy materials, both civilian and military.

Fig. 2.4 T.M. valve (Source: Musée des Arts et Métiers)

the Physics Laboratory at the *École Normale Supérieure*, and invented the *astable multivibrator* with Eugène Bloch.

2.1.3 The Multivibrator: From the Thomson-Type Systems to Relaxation Systems

Henri Abraham[13] joined the *École Normale Supérieure* at age 18, and came second in the physics agrégagtion competitive exam in 1889. The following year, after his military service, he went back to the E.N.S., as a "caiman[14]" for the Physics Laboratory, and started a thesis paper titled "Sur une nouvelle détermination du rapport v entre les unités électro-magnétiques et électro-statiques" ("On a new determination of the 'v relation between electromagnetic and electrostatic units") under the supervision of both Jules Violle and Marcel Brillouin, which he defended

[13]Henri Abraham biographies are available, such as: "À la mémoire de Henri Abraham, Eugène Bloch, Georges Bruhat: Créateurs et f Directeurs de ce Laboratoire Morts pour la France", École Normale supérieure, Physics laboratory, École Normale Supérieure, printing house Lahure, Paris, 1948.

[14]Agrégé-préparateur in ENS jargon. Term introduced in 1852.

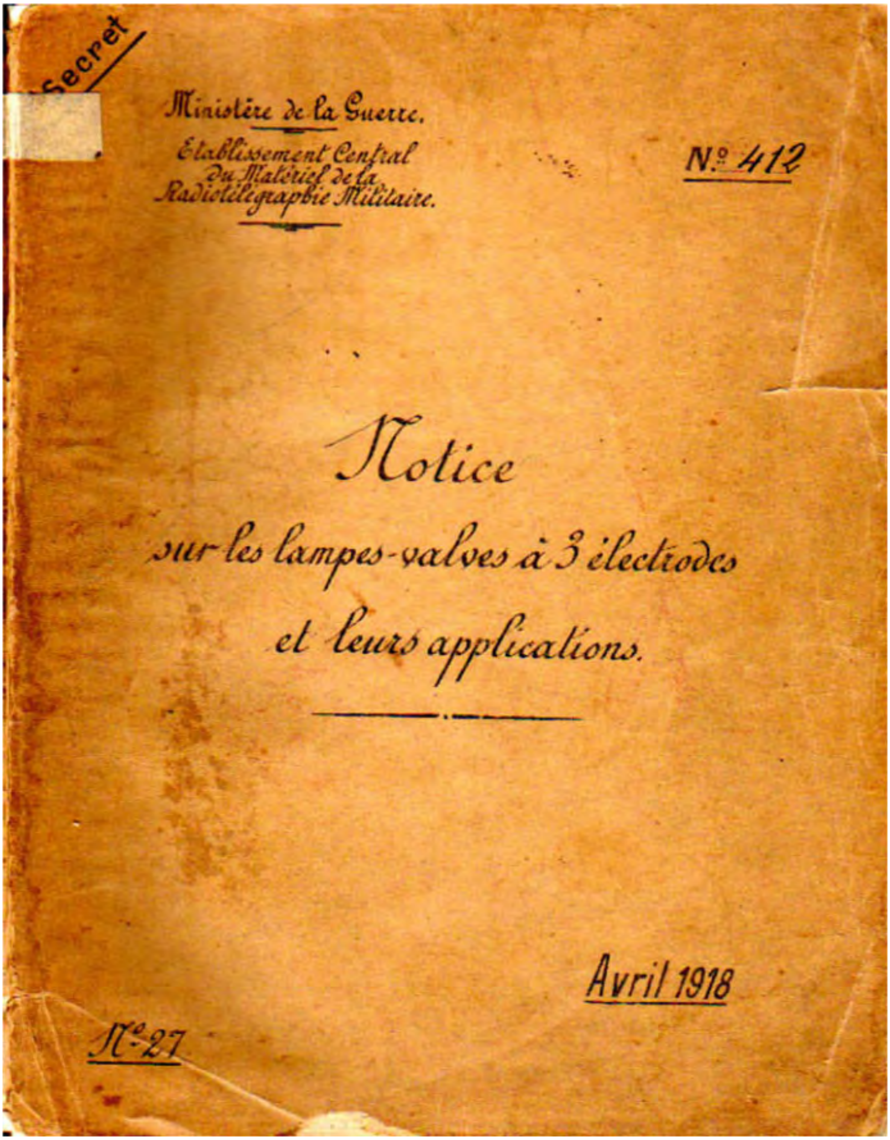

Fig. 2.5 E.C.M.R. report n° 412 (Source Jacques Denys)

barely two years later on the 30th of June 1892. He then started teaching, from 1891 to 1897, at the *lycée* Chaptal, and from 1894 to 1900 at the *lycée* Louis-le-Grand. In 1900, he was appointed associate professor at the E.N.S. then full-professor at the University of Paris. He then became head of the Physics Laboratory. After the war was declared and his six-month long stay at the Lyon-La-Doua station, he was reinstated in Paris in May 1915 and from there carried on with his work on the three-

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Fig. 2.8 E.C.M.R. report n° 412, table of contents (Source Jacques Denys)

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Fig. 2.9 E.C.M.R. report n° 412, table of contents (Source Jacques Denys)

electrode valve with Jacques Biguet.[15] At the same time, he continued his research with Eugéne Bloch for the Military Telegraphy Service, which became the E.C.M.R. two years later. In November 1917, they invented a device able to measure wireless telegraphy emitter frequencies: the *multivibrator*. They then published classified[16] Notes for the E.C.M.R. with the following titles and publication dates (Fig. 2.9):

- Sur la mesure des longueurs d'ondes de T.S.F. en valeur absolue avec le multivibrateur T.M., novembre 1917, *E.C.M.R.*, n° 2896 (On wireless telegraph wavelengths measurement in absolute value with the T.M. multivibrator),

[15] Abraham and Bloch (1920, 57) reminded that these studies were "done in collaboration with Mr. BIGUET, in 1914–1915 in the incandescent light bulb factory M. A. Grammont in Lyon". Peri's name is mentioned nowhere.

[16] The period during which Abraham and Bloch's notes were "classified" was fifty years long. They were kept until then at the *Service Historique de la Défense* (S.H.D.) in Vincennes, these notes were destroyed during a reconstruction of the premises. The Note n° 412 by Gutton was the only one found as of now. See *supra*.

- Étalonnement en valeur absolue des contrôleurs d'ondes par l'emploi du multivibrateur, novembre 1917, *E.C.M.R.*, n° 2949 (Calibration of radio wave regulators in absolute value by use of the multivibrator),
- Multivibrateur T.M. type A et type B, décembre 1917, *E.C.M.R.*, n° 2900 (T.M. multivibrator type A and type B),
- Ondes entretenues étalons. Mesure des longueurs d'ondes en valeur absolue, 5 juillet 1918, *E.C.M.R.*, n° 4448 (Sustained wave calibration. Wavelength measurements in absolute value),
- Etalonnage d'un diapason en valeur absolue, octobre 1918, *E.C.M.R.*, n° 4148, (Calibration of a tuning fork in absolute value).

After the war, Abraham and Bloch published three articles, which were public versions of the E.C.M.R. notes, and for which the title appears to have been inspired by note n° 4448.

- [1919a] Sur la mesure en valeur absolue des périodes des oscillations électriques de haute fréquence, *C.R.A.S.*, 168 (2 juin 1919), p. 1105–1108 (On high-frequency electric oscillation period measurement in absolute value),
- [1919c] Mesure en valeur absolue des périodes des oscillations électriques de haute fréquence, *J. Phys. Theor. Appl.* 9 (4 juillet 1919), p. 211–222 (High-frequency electric oscillation period measurement in absolute value),
- [1919e] Mesure en valeur absolue des périodes des oscillations électriques de haute fréquence, *Ann. de Phys.* 9 (septembre-octobre 1919), p. 237–302 (High-frequency electric oscillation period measurement in absolute value).

By replacing the classified E.C.M.R. notes, these publications consequently caused a two-year gap as to the date of invention of the multivibrator, which should have been December 1917 rather than 1919, as Abraham and Bloch recalled:

> This method was researched during the years 1916 and 1917, due to the Military Telegraphy's requirements. (Abraham and Bloch 1919a, 1106, e, 244)

> We built various types of amplifiers for the Military Telegraphy (1916). (Abraham and Bloch 1919b, 1198)

> We had to develop this measuring method during the years 1916 and 1917, by researching the causes of specific anomalies in the valve amplifiers of the military telegraphy. (Abraham and Bloch 1919c, 212)

In spite of these references to research being carried out during the war – and even though Abraham and Bloch (1919e, 244) gave a detailed list of all the E.C.M.R. reports – there were no clarifications of the patent Abraham and Bloch had just submitted that same year, 1919, for the invention of the *multivibrator*. Moreover, despite their identical titles, these publications had different contents. Paradoxically, the most complete version, published in the *Annales de Physique*, is also the less quoted. This 65-page article seems to match the E.C.M.R. reports, since the table of contents is comprised of absolutely all the titles of the original reports (see Table 2.1). Paragraph III is dedicated to the description of the multivibrator, a device (see Fig. 2.10) containing two T.M. valves, where each grid is linked to the

Table 2.1 Table of contents of Abraham and Bloch's article (1919e) vs. E.C.M.R. notes (1917–1918)

I. Introduction and principles of the method	
II. Fundamental frequency: calibration of the tuning fork	Calibration of a tuning fork in absolute value, October 1918, *E.C.M.R.*, *n*°4148
III. Production of sustained electric oscillations with many harmonics: multivibrator	T.M. multivibrator type A and type B, December 1917, *E.C.M.R.*, *n*°2900
IV. Practical implementation of the multivibrator	
V. Properties of the multivibrator	
VI. Assembly for the calibration of a wavemeter	Calibration of radio wave regulators in absolute value by use of the multivibrator, November 1917, *E.C.M.R.*, *n*°2949
VII. Action of the wavemeter on the detector-amplifier	
VIII. Mutual reactions of the various circuits. Necessity of using weak couplings	
IX. Role of the heterodyne. Octave method. Harmonics numbering	
X. Operational mode for the calibration of a wavemeter	
XI. Reading the measurements	
XII. Measurement in absolute value of the wavelength of sustained waves received by a wireless telegraph station	On the measurement of wireless telegraph wavelengths in absolute value using the T.M. multivibrator, November 1917, *E.C.M.R.*, *n*°2896 Sustained wave calibration. Wavelength measurement in absolute value. 5 July 1918, *E.C.M.R.*, *n*°4448

other's plate by a capacitor. Such an assembly produces harmonic-rich oscillations. Abraham then explained that this was the reason he called it a *multivibrator*, and described the observed phenomenon:

> The experiment being thus arranged, and the lamps being turned on, we can see that the electric currents flowing through the various circuits are subjected to abrupt periodic variations, and that there can be *no stable steady-current*. The two lamps function alternatively. At one point, the first plate suddenly starts to discharge current, whereas the second one discharges none. A few seconds later, the roles are reversed. And some more seconds later, we go back to the first flow, and so on and so forth, periodically. (Abraham and Bloch 1919e, 255)

The current flow in the second lamp (subscripts 2 on Fig. 2.10) decreases the potential in the G_1 grid, and obstructs the current flow in the first (subscripts 1 on Fig. 2.10), and vice versa. Theoretically, the device can therefore only operate in two distinct states: in which the first lamp is conducting and the second lamp has been cut-off, or the opposite. However, due to the presence of capacitors between the grids and plates of each lamp, these two operating states are unstable, and the device

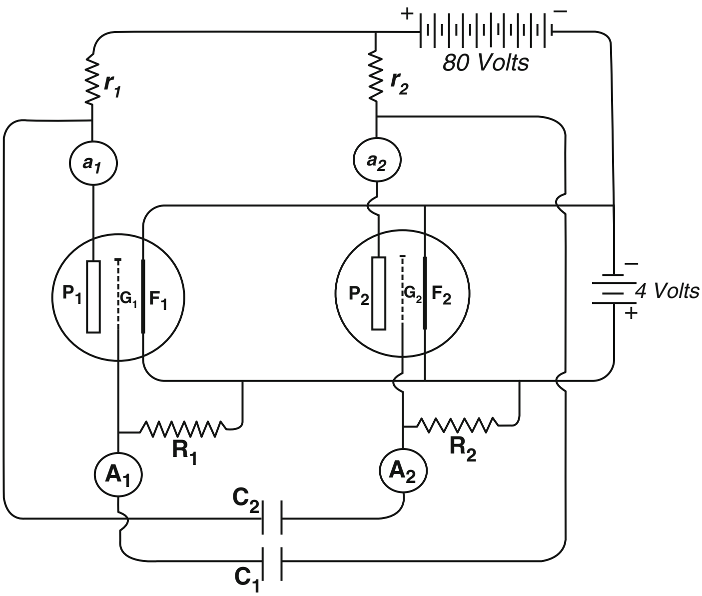

Fig. 2.10 Multivibrator, from Abraham and Bloch (1919e, 254)

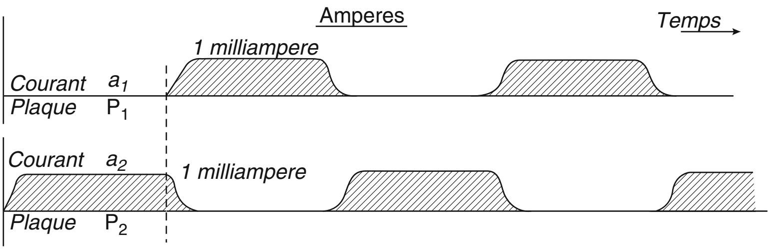

Fig. 2.11 P_1 and P_2 plate current reversals, from Abraham and Bloch (1919e, 256)

oscillates between the two, generating variations in voltage and electric current,[17] as shown on Fig. 2.11.

Meanwhile, researchers calculated the oscillation period of a circuit comprised of a capacitor of capacitance *C* and an inductor of inductance *L* by using Thomson's or Duddell's formula (see *supra* Tableau 1.1). However, Abraham demonstrated that the oscillations generated by the multivibrator have a period that does not complied with these formulae, i.e. which were not "Thomson-type". He explained that the

[17]For more details, see the pages 29–32 of the book *Les trois physiciens Henri Abraham, Eugéne Bloch, Georges Bruhat*, éditions Rue d'Ulm 2009.

observed effects on Fig. 2.11 are divided by time intervals corresponding to the charge and discharge times of capacitors C_1 and C_2 through resistors R_1 and R_2.

He deduced the following:

> The system's period is therefore of the order of $C_1R_1 + C_2R_2$. (Abraham and Bloch 1919e, 257)

This reversal time, called the "relaxation time[18]", was then used to refer to the duration of the discharge of the resistor's capacitor. As for the multivibrator, the oscillatory phenomenon is "steered" by the capacitor. The oscillation period is consequently not provided by Thomson's formula anymore, but corresponds to the "relaxation time". Through this invention, Abraham and Bloch therefore brought a new type of oscillating system to light: a relaxation system. It should be noted, however, that neither Abraham nor Bloch used this terminology, which was only introduced a few years later by Van der Pol (1925, 1926a,b,c,d) (see *infra*).

Wireless telegraphy development, spurred by the war effort, went from craft to full industrialization. The triode valves were then marketed on a larger scale. More reliable and stable than the singing arc, the consistency of the various components used in the triode allowed for exact reproduction of experiments, which facilitated research on sustained oscillations. While the singing arc itself became more and more obsolete as time passed, the properties of the oscillatory phenomenon that were discovered using this device did not. In France, Paul Janet and André Blondel set to work on transposing the different results they achieved to the triode, and their work contributed to the nonlinear oscillation theory development. Thanks to Van der Pol's work based on the multivibrator study, a new type of oscillation came into existence: *relaxation oscillations* (Figs. 2.12 and 2.13).

2.2 The Three-Electrode Valve or Triode: *Sustained Oscillations*

2.2.1 Paul Janet's Work: Analogy and Incomplete Equation Modeling (II)

In April 1919, Janet published an article of considerable importance on several levels. Firstly, it underscored the technology transfer taking place, in which an electromechanical component (the singing arc) was replaced with what would later be called an electronic tube. This represented a true revolution, since the structure of the singing arc made experiments complex, tricky, and almost impossible to recreate. Secondly, it revealed "technological analogy", based on the *duality principle*

[18] This terminology was introduced by James Clerk Maxwell (1867, 56) as reminded by Colin (1893, 1251) in a note to the *C.R.A.S.* which seemed to be one of the first occurrences of this periodic.

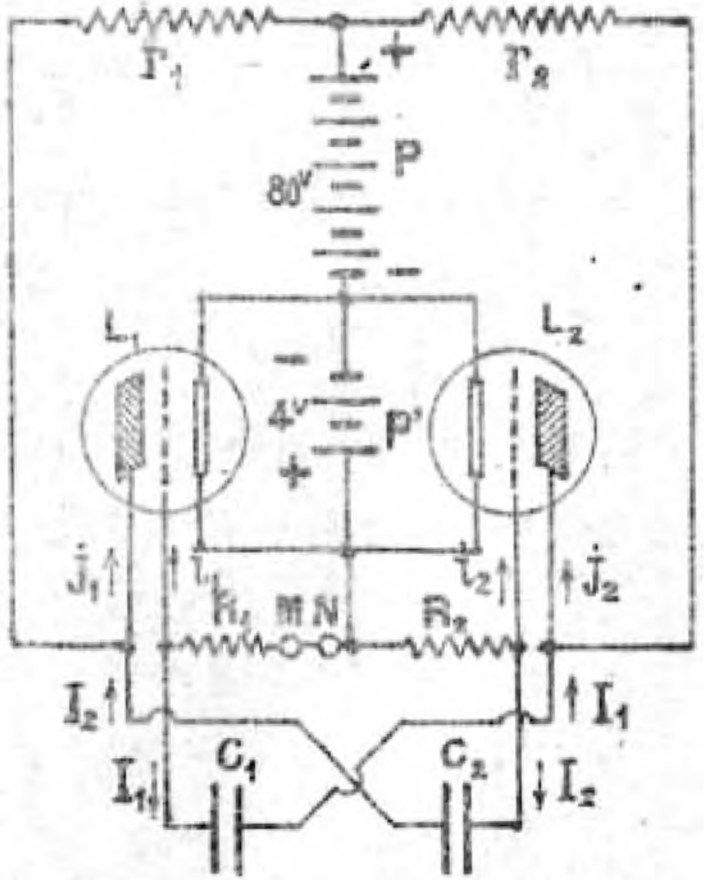

Fig. 2.12 Multivibrator, excerpt from the E.C.M.R. report n° 412 (1918) (Source Jacques Denys)

determined by Sire de Vilar (1901) between sustained oscillations produced by a series dynamo machine like the one used by Gerard-Lescuyer (1880a) and the oscillations of the singing arc or a three-electrode valve.

> I felt it was interesting to note unexpected analogies between this experiment[19] and the sustained oscillations so widely used nowadays in wireless telegraphy, for instance, those produced by Duddell's arc or the three-electrodes lamps used as oscillators. (Janet 1919, 764)

However, in this article, Janet blurs the experiment's paternity by crediting Witz (1889a). He mentioned only this "very old experiment on applied electricity, carried

[19] It is Gerard-Lescuyer's experiment (1880a).

Fig. 2.13 Multivibrator of Abraham and Bloch (Source: Musée des Arts et Métiers)

out in 1880[20]" (Janet 1925, 1193) in his article prefacing studies by the Cartans. Janet justified this "electrotechnical analogy" by basing his reasoning on an older analogy, regarding circuit components.

> Producing and sustaining oscillations in these systems mostly depends on the presence, in the oscillating circuit, of something comparable to a negative resistance. Now, the generating series-dynamo machine acts as a negative resistance, and additionally the separate excitation motor acts as a capacitor. Curiously, these two analogies have been mentioned a long time ago, the first one by M. P. Boucherot[21] and the second one by Mr. Maurice Leblanc.[22] (Janet 1919, 764)

He considered that in order to have analogies in the effects, i.e. in order to see the same type of oscillations in the series-dynamo machine, the triode and the singing arc, there must be an analogy in the causes. In fact, since the series-dynamo machine acts as a negative resistance, responsible for the oscillations, there is indeed an analogy. Consequently, one equation only must correspond to these devices. In this article, Janet appears to be the first to write the incomplete differential equation characterizing the oscillations noted during Gérard-Lescuyer's experiment:

[20] Gerard-Lescuyer (1880a).

[21] See Boucherot (1904).

[22] See Leblanc (1899).

> Writing the equation for the problem, in the case concerning the installation described above, is easy. Let $e = f(i)$ be the series-dynamo electromotive force, R and L be the resistance and self-inductance of the circuit, ω the angular velocity of the separate excitation motor. We, of course, obtain
>
> $$Ri + L\frac{di}{dt} = e - k\omega$$
>
> $$ki = K\frac{d\omega}{dt}$$
>
> hence
>
> $$L\frac{d^2i}{dt^2} + [R - f'(i)]\frac{di}{dt} + \frac{k^2}{K}i = 0 \qquad (2.1)$$
>
> (Janet 1919, 765)

The first equation, which can be rewritten as $e = L\frac{di}{dt} + Ri + k\omega$, expresses what Janet asserted (1900, 222) a few years earlier (see *supra*): in order to completely explain the phenomenon, the following must be taken into account:

(a) the *e.m.f.* of the dynamo: $e = f(i)$,
(b) the *c.e.m.f.* of the motor: $Ri + k\omega$,
(c) the *e.m.f.* of the inductor: $L\frac{di}{dt}$.

By deriving the first equation, and taking the second one into account, he easily established the last one. Then, noticing that the separate excitation machine "acts as a capacitor of capacitance K/k^2" (Janet 1919, 765), he obtained an equation perfectly analogous to the one (Van der Pol 1920, 702) established the following year for the triode. This equation was nevertheless incomplete, as also noted by Janet:

> But the phenomenon is limited by the characteristic's curvature, and regular, non-sinusoidal equations actually occur. They are governed by the equation (2.1), which could only be integrated if we knew the explicit for of the function $f(i)$. (Janet 1919, 765)

Indeed, the question of mathematical representation of the *oscillation characteristic*, i.e. the establishment of the function $f(i)$ draws on the polynomial interpolation of a curve, a concept called observable modeling nowadays. This implies the procurement of a minimal number of points, i.e. a series of facts or measurements, which requires on the one hand the exact reproducibility of the experiment, and on the other hand a measuring device able to provide accurate values. Janet therefore demonstrated, by establishing an analogy between three different devices, that they all fell under the same oscillatory phenomenon, for which he provided the general, albeit incomplete, equation. It is, however, surprising that Janet did not refer Poincaré's work (1908) on the singing arc, even though he cited the older studies conducted by Leblanc (1899) and Boucherot (1904). Nevertheless, in his note, Janet also explained the main obstacle to overcome in order to complete his equation. In November of the same year, 1919, Blondel was the one who solved the problem by establishing, one year before Van der Pol, the equation for the triode and introducing the term "self-sustained oscillations" in order to qualify the phenomenon.

2.2.2 *André Blondel: The Anteriority of the Writing of the Triode Equation*

After he fully solved the question concerning the nature of the electric arc and demonstrated that it does not possess a *c.e.m.f.*, Blondel tackled the oscillatory phenomenon occurring in the singing arc. From 1919, during a transition from the singing arc to the triode, he began his research "by analogy with the already established theory on the singing arc" (Blondel 1919a, 676), transposing most of the results he had obtained.

2.2.2.1 Modeling

As stated by Poincaré (1908), then Janet (1919), the only obstacle preventing the formulation of a complete equation for the oscillations observed in the triode, the singing arc, and Gérard-Lescuyer's experiment, was the modelling of the oscillation characteristic of the nonlinear component, comparable to a negative resistance, present in the three devices. In 1919, Blondel was the first to model the nonlinear characteristic of the triode, using a development "in the form of an uneven terms series" (Blondel 1919b, 946). He therefore established his differential equation a year before Van der Pol (1920). Philippe Le Corbeiller (see *infra* Part II) was the one to point out this apparently unheard-of result.

> Mr. A. Blondel, in 1919, researched in this manner[23] the oscillating triode shown on Fig. 6. (...) and found a third order equation (...)

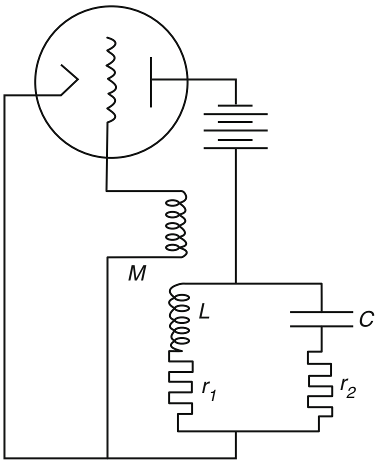

Fig. 6.

[23] See Blondel (1919b).

> In 1920, M. van der Pol also studied this same oscillating triode diagram, but by constructing the oscillating plate circuit with a pure inductance L, a capacitance C and a resistance R in parallel, hence obtaining a second order equation. (Le Corbeiller 1932, 705)

In a note published in the *C.R.A.S.* and presented at the *Académie des Sciences* on the 17th of November 1919, Blondel modeled the oscillation characteristic of the triode. By calling u the voltage in the plate with i being the variation in the plate current of the triode, v the grid potential and k the amplification coefficient of the triode, he explained the relation between u and i in the triode, formed by $i = F(u + kv)$ and "in which F represents a function reflected by a known curve presenting a long inflection around the average value of the static current I (approximately equal to half the value of the saturation current)." (Blondel 1919b, 946). He then hypothesized that we stay in the area where this curve keeps the same form regardless of the value of v and moves only in parallel with itself, by a translation along the u axis when v varies. This led him to model the oscillation characteristic $i = F(u + kv)$ of the triode, and he offered to "develop it as a series of odd terms which must be convergent" (Blondel *Ibid.*, 946). He obtained:

$$i = F(u + kv) = b_1(u + kv) - b_3(u + kv)^3 - b_5(u + kv)^5 - \dots \tag{2.2}$$

2.2.2.2 Writing the Equation

This note titled "Amplitude du courant oscillant produit par les audions générateurs" ("The oscillating current amplitude produced by generating audions") aimed to calculate an approximation of the amplitude of the oscillations. For this reason, Blondel established the triode's equation and introduced the modeling of its characteristics. Then, by calling i the plate current intensity at the moment t, i_1 and i_2 the intensities in the branches of self-inductance L and capacitance C, with internal resistances $r_1$and r_2 respectively, u the oscillating voltage at the parallel circuit terminals, he obtained the three following equations:

$$i_1 + i_2 = i \quad , \quad r_1 i_1 + L\frac{di_1}{dt} = u \quad , \quad r_2 i_2 + \frac{1}{C}\int i_2 dt = u \quad , \quad h = \frac{kM}{L} - 1;$$

By combining and deriving them, Blondel (1919b, 945) established the triode's differential equation with the form:

$$\frac{d^3u}{dt^3} + \frac{r_2}{L}\frac{d^2u}{dt^2} + \left(\frac{1}{CL} - \frac{r_1 r_2}{L^2}\right)\frac{du}{dt} - \frac{r_1}{CL^2}u - r_2\frac{d^3 i}{dt^3} - \frac{1}{C}\frac{d^2 i}{dt^2} = 0 \tag{2.3}$$

The presence of internal resistances r_1 and r_2, which Blondel could have neglected, led him to this third order quadratic differential equation. By substituting the expression (2.2) of the intensity i in this equation (2.3), "The final equation for the problem" (Blondel 1919b, 947) took the following form:

$$
\begin{aligned}
&\frac{d^3u}{dt^3} + \frac{d^2u}{dt^2}\frac{r_2}{L} + \frac{du}{dt}\left(\frac{1}{CL} - \frac{r_1 r_2}{L^2}\right) - \frac{r_1}{CL^2}u \\
&-\left(r_2\frac{d^2u}{dt^2} - \frac{1}{C}\frac{d^2u}{dt^2}\right)\left[b_1\left(h - \tfrac{kMr_1}{L^2}u\right) - 3b_3h^3u^2 - \ldots\right] \\
&-\left[3r_2\frac{d^3u}{dt^3}\frac{du}{dt} + \frac{1}{C}\left(\frac{du}{dt}\right)^2\right]\left[-\frac{b_1kMr_1}{L^2} - 6b_3h^3u + \ldots\right] \\
&-r_2\left(\frac{du}{dt}\right)^3\left[-6b_3h^3 - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\right] = 0
\end{aligned}
\tag{2.4}
$$

Neglecting the internal resistances r_1 and r_2, i.e. posing in equation (2.4): $r_1 = r_2 = 0$, we have:

$$
\frac{d^3u}{dt^3} + \frac{du}{dt}\left(\frac{1}{CL}\right) + \frac{1}{C}\frac{d^2u}{dt^2}\left[b_1h - 3b_3h^3u^2 - \ldots\right] - \frac{1}{C}\left(\frac{du}{dt}\right)^2\left[-6b_3h^3u + \ldots\right] = 0
$$

Grouping the terms in b_3h^3, we obtain:

$$
\frac{d^3u}{dt^3} + \frac{du}{dt}\left(\frac{1}{CL}\right) + \frac{1}{C}\frac{d^2u}{dt^2}(b_1h) - \frac{3b_3h^3}{C}\left[\frac{d^2u}{dt^2}u^2 + \left(\frac{du}{dt}\right)^2(2u) + \ldots\right] = 0
$$

And noticing that the last term is written as:

$$
\left[\frac{d^2u}{dt^2}u^2 + \left(\frac{du}{dt}\right)^2(2u) + \ldots\right] = \frac{d}{dt}\left(\frac{du}{dt}u^2\right) + \ldots
$$

then integrating once in relation to time, we obtain for this equation (2.4):

$$
C\frac{d^2u}{dt^2} - \left(b_1h - 3b_3h^3u^2 - \ldots\right)\frac{du}{dt} + \frac{u}{L} = 0 \tag{2.5}
$$

In the same manner, by directly integrating the equation (2.3), we would have obtained:

$$
C\frac{d^2u}{dt^2} - \frac{di}{dt} + \frac{1}{L}u = 0 \tag{2.6}
$$

2.2.2.3 Calculating the Fundamental's Period and Amplitude

By developing the voltage u in the equation (2.4) in a Fourier series, and identifying it term by term, Blondel deduced a first approximation of the period (angular frequency, or pulsation) ω for the oscillations:

$$\omega^2 \approx \frac{1}{CL}\left(1 + \frac{r_2}{\rho}h_0 - r_1 r_2 \frac{C}{L}\right) \text{ with } h_0 = \rho C \frac{r_1 + r_2}{L} \tag{2.7}$$

If we neglect the internal resistances r_1 and r_2 again, this expression is reduced as follows:

$$\omega^2 \approx \frac{1}{CL} \tag{2.8}$$

It should be noted that Blondel found Thomson's formula (1853) as a first approximation (see *supra*). Using the same technique, Blondel obtained a first approximation of the oscillation amplitude A_1:

$$A_1 \approx \frac{2}{h}\sqrt{\frac{-b_1 + \frac{r_1 + r_2}{hL}C}{-3b_3}} \tag{2.9}$$

If we neglect the internal resistances r_1 and r_2 again, this expression is reduced as follows:

$$A_1 \approx \frac{2}{h}\sqrt{\frac{b_1}{3b_3}} \tag{2.10}$$

In June the following year, 1920, Blondel explained these results in a longer and more detailed article. Before that, he looked into the origin of the phenomenon, and offered a classification of various types of oscillations.

2.2.2.4 Classifying Oscillations

In an article published in 1919, in which Blondel suggests a classification comprised of three main categories of oscillations, he is the first to introduce the term self-sustained oscillations. There are no traces of this neologism prior to this publication,[24] which he appears to have formed by associating "self-started sustained oscillations" (Blondel 1919d). In his classification, the first type includes "oscillations sustained by continuous action, or self-sustained oscillations" (Blondel 1919d, 118), with notable examples such as Duddell's singing arc and the triode. The second type, which covers the "divided flow" oscillations, is illustrated by the Tantalus vase, or cup, and the vase culbuteur ("tumbler vase"). However, it is the third type that represents the "long-period oscillations" (Blondel 1919d, 124) which he linked, in reference to Janet, to Gérard-Lescuyer's experiment, rather than the first type:

[24]Nevertheless, as soon as the end of the twentieth century, the *series-dynamo machine* was already called *self-exciting*. See *supra*.

> It seems to me that the electric oscillations produced by the excitation's reversal in a generator, such as the series-excited dynamo, should be classified under this same type (long-period oscillation). Mr. Janet recently mentioned a type of oscillation of this kind once again. (Blondel 1919c, 125)

This is especially surprising since Blondel, with this classification, seemed to have separated the series-dynamo machine from the singing arc, whereas Janet had linked them with an analogy. Although the term "self-sustained oscillations" introduced by Blondel appears to have been initially used in a slightly too constraining way, Andronov's article (1929a) greatly broadens its scope (see *infra* Part II).

2.3 Balthasar Van der Pol's *Equation for the Triode*

In 1916, following his studies in physics and mathematics at the University of Utrecht, Balthasar Van der Pol (1889–1959) went to study under John Ambrose Fleming, an English electrical engineer and physicist, who taught at University College in London. At the time Fleming was the first electrotechnics teacher there, but he was better known as the inventor of the diode, i.e. the first thermionic valve, in 1904. On the 2nd of June 1917, Van der Pol married Pietronetta Posthuma in London (see *infra* Part II) with whom he had a son and two daughters. Then, after spending one year with Fleming, Van der Pol started working with John Joseph Thomson at the Cavendish laboratory of Cambridge. In 1920, he went back to Holland in order to finish his physics doctorate at Utrecht, under Hendrik Lorentz. His dissertation addressed "the influence of ionized gas on the propagation of electromagnetic waves, as applied to wireless telegraphy and ultraviolet radiation measurement". As early as the 1920s, Van der Pol set to work on the production of electromagnetic waves, by using oscillating electric circuits containing a triode instead of a singing arc. However, as recalled by Cartwright (1960, 370) as well as Stumpers (1960, 366), it was not in his famous contribution "On relaxation-Oscillations" (Van der Pol 1926c), but in a previous article, completed[25] on 17 July 1920, and published in November and December of the same year, that Van der Pol (1920) modeled characteristic oscillation of the triode by using a cubic function, and established his differential equation one year after Blondel (1919b).

[25]The footnote in which Van der Pol (1920, 702) referred to an article written by W. E. Eccles published in November 1919 seemed to indicate that he only started the writing after this date.

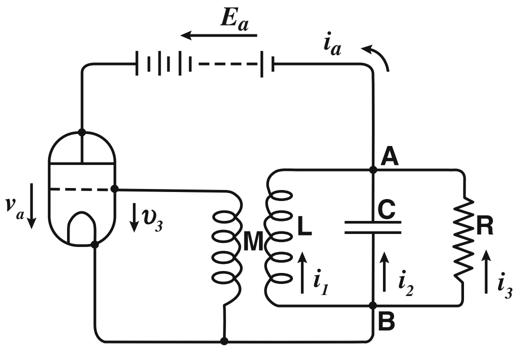

Fig. 2.14 Diagram of the oscillating triode, from Van der Pol (1920, 701)

2.3.1 Modeling

It can be observed in Fig. 2.14 that (Van der Pol 1920, 701) assembled the circuit differently to Blondel (1919b) . In order to simplify the problem as much as possible, he chose to place the internal resistances of the inductor L and the capacitor C, not in series as it should be, but in parallel, with a resistance R. This choice, justified at the start of the article by Van der Pol, led him to a second-order differential equation:

> When the non-linear terms are retained in the equations the latter, and still more their solutions, soon become very complicated and in order to show clearly and definitely the importance of these terms it seems advisable to treat analytically that system of connections which renders the equations as simple as possible, thus obviating as far as possible, purely analytical complications, and allowing the physical meaning of the formulae to be clearly seen. This is especially the case in locating the resistance of the oscillatory L C flywheel circuit connected to the anode and filament, not in series either which the induction or capacitance but in parallel to both. (Van der Pol 1920, 701)

He called v_a the voltage of the plate corresponding to the variation i_a of the anode current, i.e. the plate current of the triode, v_g the grid potential, and g the amplification coefficient of the triode. He explained the relation between v_a and i_a in the triode, with the form $i_a = \varphi\left(v_a + gv_g\right)$. He then considered that with an unstable stationary state, the plate voltage is reduced to a value $v_{a_0} = E_a$, where E_a represents the electromotive force of the generator, the current intensity in the anode therefore being $i_{a_0} = \varphi\left(v_{a_0}\right)$. He then wrote $v = v_a - v_{a_0}$ and $i = i_a - i_{a_0} = \varphi\left(v_{a_0} - kv\right) - \varphi\left(v_{a_0}\right) = \psi\left(kv\right)$, where v and i represent respectively the instantaneous voltage and intensity in the triode's plate. By using a Taylor-McLaurin series expansion limited to the first three terms, he wrote that $\psi\left(kv\right)$ "can be represented by the equation" (Van der Pol 1920, 703):

$$i = \psi\left(kv\right) = -\alpha v + \beta v^2 + \gamma v^3 \tag{2.11}$$

Van der Pol added that, using symmetry considerations for the oscillation characteristic, this expression can be reduced by writing: $\beta = 0$, as noted by Cartwright (1960, 370). Two years later, in order to describe the oscillation hysteresis phenomenon in the triode, Appleton and Van der Pol (1922, 182) had to expand the function $\psi(kv)$ to the fifth order, just as Blondel had done (1919b, 946).

2.3.2 Writing the Equation

Van der Pol's aim (1920) in this article was more ambitious than Blondel's (1919b), since he offered not only to calculate an approximation for the free oscillations of the triode, but also for the forced oscillations, as indicated by the title "A theory of the amplitude of free and forced triode vibrations". He expressed the voltage at the terminals of each dipole: $L\frac{di_1}{dt} = Ri_3 = \frac{1}{C}\int i_2 dt = E_a - v_a$ and managed to establish the differential equation for the triode:

$$\frac{di}{dt} + C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0 \tag{2.12}$$

By substituting the expression (2.11) of the intensity i in this equation (2.12), the equation for the triode that Van der Pol obtained is written as follows:

$$C\frac{d^2v}{dt^2} + \left(\frac{1}{R} - \alpha\right)\frac{dv}{dt} + \frac{1}{L}v + \beta\frac{d\left(v^2\right)}{dt} + \gamma\frac{d\left(v^3\right)}{dt} = 0$$

In order to enable a comparison with Blondel's works (1919b), we should put $\beta = 0$ and the resistance R should be disregarded by letting $R \to \infty$:

$$C\frac{d^2v}{dt^2} - \left(\alpha - 3\gamma v^2\right)\frac{dv}{dt} + \frac{1}{L}v = 0 \tag{2.13}$$

2.3.3 Calculating the Period and Amplitude of the Oscillations

In order to calculate the amplitude, Van der Pol offers three methods. The first is "analytical", as he explained (1920, 704), and consists in a singular perturbation expansion used by astronomers. The second resorts to a Fourier series expansion, used by Blondel (1919b), which was generally used by engineers, and led him to the introduction of a first correction for the value of the period (angular frequency, or pulsation ω):

$$\omega^2 \approx \frac{1}{CL} - \varepsilon \quad \text{avec} \quad \varepsilon = \frac{a^2\beta^2}{3C^2}$$

Table 2.2 Grid view of the simplified results by Blondel (1919b) and Van der Pol (1920)

Blondel (1919b)	Van der Pol (1920)
$i = F(u + kv) = b_1(u + kv) - b_3(u + kv)^3$	$i = \psi(kv) = -\alpha v + \gamma v^3$
$C\frac{d^2u}{dt^2} - (b_1 h - 3b_3 h^3 u^2)\frac{du}{dt} + \frac{u}{L} = 0$	$C\frac{d^2v}{dt^2} - (\alpha - 3\gamma v^2)\frac{dv}{dt} + \frac{1}{L}v = 0$
$\omega^2 \approx \frac{1}{CL}$	$\omega^2 \approx \frac{1}{CL}$
$A_1 \approx \frac{2}{h}\sqrt{\frac{b_1}{3b_3}}$	$a = \sqrt{\frac{4}{3}\frac{\alpha}{\gamma}}$

But since the symmetry shows that $\beta = 0$, Van der Pol (1920, 705) therefore obtained:

$$\omega^2 \approx \frac{1}{CL} \tag{2.14}$$

It should be noted that Van der Pol found, as Blondel did, the Thomson formula (1853) as a first approximation (see *supra*).

The third calculation method for the amplitude is geometrical, and apparently based on Witz's construction (1889b), which allowed him, similarly to the two previous ones, to find the following expression

$$a = \sqrt{\frac{4}{3}\frac{\alpha - \frac{1}{R}}{\gamma}} \tag{2.15}$$

By taking all the previously described simplifications into account, it is possible to establish a comparison between the studies accomplished by Blondel (1919b) and Van der Pol (1920), presented in the grid view below (see Table 2.2).

It therefore clearly appears that, by performing simplifications, the equations (2.11), (2.12), (2.13) and (2.14) written by Van der Pol (1920) and (2.2), (2.5), (2.6) and (2.8) by Blondel (1919b) are perfectly identical, the exception being the arbitrarily chosen signs representing the current.